

Name:

Solutions

Solve each equation, which has either irrational zeros or Imaginary Zeros or both:

1. Solve $x^2 + 16 = 0$

$$\begin{aligned}x^2 &= -16 \\ \sqrt{x^2} &= \pm \sqrt{-16} \\ x &= \pm i\sqrt{16} \\ x &= \pm 4i\end{aligned}$$

$$\{4i, -4i\}$$

2. Solve $y^2 + 9 = 0$

$$\begin{aligned}y^2 &= -9 \\ \sqrt{y^2} &= \pm \sqrt{-9} \\ y &= \pm i\sqrt{9} \\ y &= \pm 3i\end{aligned}$$

$$\{3i, -3i\}$$

3. Solve $x^2 - 10 = 0$

$$\begin{aligned}x^2 &= 10 \\ \sqrt{x^2} &= \pm \sqrt{10} \\ x &= \pm \sqrt{10}\end{aligned}$$

$$\{\sqrt{10}, -\sqrt{10}\}$$

4. Solve $k^2 - 12 = 0$

$$\begin{aligned}k^2 &= 12 \\ \sqrt{k^2} &= \pm \sqrt{12} \\ k &= \pm \sqrt{4 \cdot 3} \\ k &= \pm 2\sqrt{3}\end{aligned}$$

$$\{2\sqrt{3}, -2\sqrt{3}\}$$

5. Solve $p^2 + 2p - 1 = 0$

$$p = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-1)}}{2(1)}$$

$$p = \frac{-2 \pm \sqrt{4+4}}{2}$$

$$p = \frac{-2 \pm \sqrt{8}}{2}$$

$$p = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$p = -1 \pm \sqrt{2}$$

$$\{-1+\sqrt{2}, -1-\sqrt{2}\}$$

6. Solve $w^2 - 4w + 5 = 0$

$$w = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$w = \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$w = \frac{4 \pm \sqrt{-4}}{2}$$

$$w = \frac{4 \pm 2i}{2}$$

$$w = 2 \pm i$$

$$\{2+i, 2-i\}$$

7. Solve $m^3 - 1 = 0$

$$(m-1)(m^2+m+1)$$

$$m-1=0$$

$$m=1$$

$$m = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$m = \frac{-1 \pm i\sqrt{3}}{2}$$

$$m = \frac{-1 \pm \sqrt{-3}}{2}$$

$$\left\{1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$$

8. Solve $k^3 + 64 = 0$

$$(k+4)(k^2-4k+16)$$

$$(k+4)(k^2-4k+16)$$

$$k+4=0$$

$$k=-4$$

$$k = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)}$$

$$k = \frac{4 \pm \sqrt{16-64}}{2}$$

$$k = \frac{4 \pm \sqrt{-48}}{2}$$

$$k = \frac{4 \pm i\sqrt{16 \cdot 3}}{2}$$

$$k = \frac{4 \pm 4i\sqrt{3}}{2}$$

$$\{-4, 2+2i\sqrt{3}, 2-2i\sqrt{3}\}$$

9. Solve $8x^3 - 24x = 0$

$$8x(x^2-3)=0$$

$$8x=0 \quad x^2-3=0$$

$$x^2=3$$

$$x = \pm\sqrt{3}$$

$$\{0, \sqrt{3}, -\sqrt{3}\}$$

10. Solve $5m^3 + 45m = 0$

$$5m(m^2+9)=0$$

$$5m=0 \quad m^2+9=0$$

$$m=0 \quad m^2=-9$$

$$m = \pm\sqrt{-9}$$

$$m = \pm 3i$$

$$\{0, 3i, -3i\}$$

11. Solve $2x^2 + 12x + 14 = 0$

$$2(x^2+6x+7)=0$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36-28}}{2}$$

$$x = \frac{-6 \pm \sqrt{8}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{2}}{2}$$

$$\{-3+\sqrt{2}, -3-\sqrt{2}\}$$

12. Solve $x^3 + 3x^2 + 4x + 12 = 0$

$$x^2(x+3) + 4(x+3) = 0$$

$$(x+3)(x^2+4) = 0$$

$$x+3=0$$

$$x = -3$$

$$x^2+4=0$$

$$x^2 = -4$$

$$x = \pm\sqrt{-4}$$

$$x = \pm i\sqrt{4}$$

$$x = \pm 2i$$

$$\{-3, 2i, -2i\}$$

13. Solve $y^3 + 2y^2 - 7y - 14 = 0$

$$y^2(y+2) - 7(y+2) = 0$$

$$(y+2)(y^2-7) = 0$$

$$y+2=0$$

$$y = -2$$

$$y^2-7=0$$

$$y^2 = 7$$

$$y = \pm\sqrt{7}$$

$$\{-2, \sqrt{7}, -\sqrt{7}\}$$

14. Solve $p^4 + 5p^2 + 4 = 0$

$$(p^2+1)(p^2+4) = 0$$

$$p^2+1=0$$

$$p^2 = -1$$

$$p = \pm\sqrt{-1}$$

$$p^2+4=0$$

$$p^2 = -4$$

$$p = \pm\sqrt{-4}$$

$$p = \pm i \quad p = \pm 2i$$

$$\{i, -i, 2i, -2i\}$$

15. Solve $z^4 - 8z^2 + 15 = 0$

$$(z^2-3)(z^2-5) = 0$$

$$z^2-3=0$$

$$z^2 = 3$$

$$z = \pm\sqrt{3}$$

$$z^2-5=0$$

$$z^2 = 5$$

$$z = \pm\sqrt{5}$$

$$\{\sqrt{3}, -\sqrt{3}, \sqrt{5}, -\sqrt{5}\}$$

Write an equation (in standard form) of the polynomial function with the given zeros:

16. The solution set is $\{i, -i\}$, and the polynomial has degree 2

$$x = i$$

$$x - i = 0$$

$$x = -i$$

$$x + i = 0$$

$$(x-i)(x-(-i)) = 0$$

$$(x-i)(x+i) = 0$$

$$x^2 + ix - ix - i^2 = 0$$

$$x^2 - (-1) = 0$$

$$x^2 + 1 = 0$$

17. The solution set is $\{\sqrt{10}, -\sqrt{10}\}$, and the polynomial has degree 2

$$\begin{aligned}x &= \sqrt{10} & x &= -\sqrt{10} \\x - \sqrt{10} &= 0 & x + \sqrt{10} &= 0 \\(x - \sqrt{10})(x + \sqrt{10}) &= 0 \\x^2 + \sqrt{10}x - \sqrt{10}x - \sqrt{100} &= 0 \\x^2 - 10 &= 0\end{aligned}$$

18. The solution set is $\{1, \sqrt{6}, -\sqrt{6}\}$, and the polynomial has degree 3

$$\begin{aligned}x &= 1 & x &= \sqrt{6} & x &= -\sqrt{6} \\x - 1 &= 0 & x - \sqrt{6} &= 0 & x + \sqrt{6} &= 0 \\(x - 1)(x - \sqrt{6})(x + \sqrt{6}) &= 0 \\(x - 1)(x^2 + \sqrt{6}x - \sqrt{6}x - \sqrt{36}) &= 0 \\(x - 1)(x^2 - 6) &= 0 & x^3 - x^2 - 6x + 6 &= 0\end{aligned}$$

19. The solution set is $\{0, 3i, -3i\}$, and the polynomial has degree 3

$$\begin{aligned}x &= 0 & x &= 3i & x &= -3i \\x &= 0 & x - 3i &= 0 & x + 3i &= 0 \\x(x - 3i)(x + 3i) &= 0 \\x(x^2 + 3ix - 3ix - 9i^2) &= 0 \\x(x^2 - 9(-1)) &= 0 & x^3 + 9x &= 0\end{aligned}$$

20. The solution set is $\{\sqrt{2}, -\sqrt{2}, -i\sqrt{5}, i\sqrt{5}\}$, and the polynomial has degree 4

$$\begin{aligned}x &= \sqrt{2} & x &= -\sqrt{2} & x &= -i\sqrt{5} & x &= i\sqrt{5} \\x - \sqrt{2} &= 0 & x + \sqrt{2} &= 0 & x + i\sqrt{5} &= 0 & x - i\sqrt{5} &= 0 \\(x - \sqrt{2})(x + \sqrt{2})(x + i\sqrt{5})(x - i\sqrt{5}) &= 0 \\(x^2 - \sqrt{4})(x^2 - i^2\sqrt{25}) &= 0 \\(x^2 - 2)(x^2 - (-1)(5)) &= 0 \\(x^2 - 2)(x^2 + 5) &= 0 \\x^4 + 5x^2 - 2x^2 - 10 &= 0 \\x^4 + 3x^2 - 10 &= 0\end{aligned}$$